Title: Growing Patterns - How Many Bumps on a Pineapple? - A Lesson in Growing Patterns Featuring Fibonacci Sequence

Brief Overview:

Students begin this concept development lesson by copying pictorial patterns using snap cubes and progress to continuing these patterns by adding the next term in the sequence. Students also create patterns of their own and members of their cooperative groups describe the pattern and determine the rule. The lesson moves into putting patterns and number sequences into function tables and explores determining the rule of a number sequence using a function table. This mathematics lesson is connected to science through Fibonacci sequence. The students learn that Fibonacci sequence is a number sequence that is also found in nature.

NCTM Content Standard/National Science Education Standard:

Algebra

The students should be able to:

- Understand patterns, relations, and functions

Science as Inquiry

All students should develop:

- Abilities necessary to do scientific inquiry
- Understandings about scientific inquiry

Grade/Level:

Grade 5

Duration/Length:

3 days
Lesson 1 (60 minutes)
Lesson 2 (60 minutes)
Lesson 3 (90 minutes)

Student Outcomes:

Students will:

- Interpret and write a rule for a one operation function table.
- Create a one operation function table to solve a real world problem.

Materials and Resources:

- Pineapple(s)
- Pinecones (one per student)
- Sunflower(s)
- White Out Pens
- Numbered Pinecone
- Snap Cubes
- Overhead Pattern Blocks
- Predicting Patterns (SR1)
- Brief Constructed Response (SR2)
- Brief Constructed Response Answer Sheet (TR1)
- Homework (SR3)
- Homework Answer sheet (TR2)
- Brief Constructed Response (SR4)
- Brief Constructed Response Answer Sheet (TR3)
- Homework (SR 5)
- Homework Answer Sheet (TR4)
- Brief Constructed Response (SR6)
- Brief Constructed Response Answer Sheet (TR5)
- Building Patterns (TR6)
- Background on Fibonacci (TR7)
- Picture of Segmented Pinecone (TR8)
- Pineapple Rows (TR9)
- Sunflower Spirals (TR10)
- MSA Rubric (TR11)`

Development/Procedures:

Lesson 1

Preassessment –

- Give each student 40 snap cubes. You may use any other manipulative such as blocks, cheerios, or paper clips.
- Write the growing number pattern 1, 3, 5, 7 where it is visible to all students. Ask students to copy this pattern using their manipulatives by representing the number one with one snap cube, the number three with three snap cubes, and so on.
- Model representing the number pattern using overhead pattern blocks. Assess if students understand what the pattern is by asking them what number comes next.
- Elicit responses from your students that describe the pattern by asking: What do you notice about each number? What is the difference between each number? Is each number increasing by the same amount?
- Repeat this process with the number patterns 1, 4, 7, 10 and 1, 2, 4, 7, 11.

Launch –

- Show students the pineapple, pinecone, and sunflower. Pass these items around the room allowing the students to touch them.
- Ask students to describe the items and create a process chart listing the descriptions of each item. Most likely the students will mention the bumps on the pineapple, the segments on the pinecone, and the seeds on the sunflower. If not, elicit these descriptions from your students by asking what they feel when they touch the pineapple, does the pinecone have different parts, and what are in the center of the sunflower.
- Explain that you are beginning a unit on growing patterns. Ask students if they can believe that by the end of the unit they will be able to figure out how many bumps are on the pineapple, how many segments are on the pinecone, and how many seeds are in the sunflower without counting them one by one.

Teacher Facilitation –

- Make the growing pattern 2, 4, 6, 8 with overhead pattern blocks. Ask students to copy the pattern that you have made.
- Ask students to continue the pattern by predicting the next amount of pattern blocks or the next term.
- At this time you should introduce your students to the vocabulary terms **term** and **sequence**. You may use a "Math Talk" chart to keep track of the vocabulary terms students encounter during this unit. Simply label a piece of chart paper with the words "Math Talk". Post it somewhere students will be able to reach. When a student encounters a new vocabulary word, he/she can record the words on the chart him/herself.
- Ask the class if anyone has ever heard of the words term or sequence. Ask any volunteers to explain what the word means to the class. Explain that a **term is each place or position in the sequence** and a **sequence is a set of objects or numbers arranged in a special order or pattern.**
- Elicit responses from your students that describe the pattern by asking: What is the difference between each term?, Is the amount of difference between each term in the sequence the same?, and How do you know which number will come next in the sequence?
- Repeat this process with the number pattern 4, 7, 10, 13
- Ask students to use their manipulatives to build a pattern when given a description. For example, ask your students to build a pattern that grows by three.
- Next ask students to build a pattern that doubles the previous number. Then ask students to build a pattern that increases by 4. Refer to TR6 to see what the patterns should look like.

Student Application –

- Explain that students are going to be creating patterns with their manipulatives. Then asking their group members to represent the patterns with numbers, determine the pattern, and predict the next number in the pattern. Then the group members will be switching back to determine if their partner identified the pattern that the pattern maker had in mind.

- Distribute SR1. Ask each student to create a growing pattern using their manipulatives. Give students a few minutes to create their patterns and then ask them to determine their partner's pattern and make their prediction on SR1.
- If the pattern maker had a different pattern in mind than his/her partner describes, but they both are correct, then take that opportunity to discuss that it is possible for both students to be correct.

Embedded Assessment –

- Distribute SR2. Ask each student to predict the next number in the pattern on SR2 and to explain how they were able to determine the next number in a brief constructed response. Answer key may be found on TR1.

Reteaching–

- Display the number pattern 5, 10, 15 and ask students to copy the pattern using their manipulatives.
- Ask the students to continue and describe the pattern.
- Ask the students to build a pattern that grows by 2 with their manipulatives.

Extension-

- Show students the number sequence 4 8 _ 16 20. Ask students to determine what number is missing. Elicit responses by asking: What is the pattern between the numbers that are present?
- Distribute SR3 to be completed for homework. Answer key may be found on TR2.

<u>Lesson 2</u>

Preassessment –

- Ask students to take out SR3 and review the answers. Encourage students to explain how they arrived at the answer.
- Show students the number sequence 6, 8, __, 12, 14. Ask students to determine what number is missing.
- Show students the number sequence 3, 9, __, 81. Ask students to determine what number is missing.
- Show students the number sequence 3, 5, 7, 8, 9, 11. Ask students to determine which term does not belong.
- Show students the number sequence 2, 4, 8, 16, 20, 32. Ask students to determine which term does not belong.

Launch-

- Ask students if they have ever heard of Fibonacci or Leonardo of Pisa. Explain that Fibonacci was the first person to investigate a sequence of numbers, which we will learn more about later.

- Add that the sequence of numbers that Fibonacci investigated is related to how we will determine the bumps on the pineapple, the segments on the pinecone, and the seeds on the sunflower tomorrow.
- Read TR7 aloud to the students.

Teacher Facilitation-

- Distribute 20 snap cubes to each student
- Display pattern blocks that represent the number sequence 4, 5, 6.
- First ask students to copy the pattern using their snap cubes.
- Ask students to represent the pattern in a number sequence. After observing students, write the number sequence 4, 5, 6 on the overhead.
- Explain to students that they can use a function table to help them easily determine the next number in the sequence. Explain that a **function is a set of ordered pairs such that for any first number (the term), there is only one possible second number (the number)**. Remind students about the “”Math Talk” chart.
- Draw a function table on the overhead. Explain that the terms are to be placed in the left column. Remind the students that a **term is each place or position in the sequence**. Remind students that a **sequence is a set of objects or numbers arranged in a special order or pattern**.
- Label the left column “terms”.
- Explain that the numbers in the sequence are to be placed in the right column and label the right column “numbers”.
- Next explain that 4 is the first term in the number sequence because it comes first. Ask students what is the second term in the number sequence. Ask students which term the number 6 is.
- Explain that you are going to write 1 in the “terms” column and then you are going to write 4 in the “numbers” column because four is the first term in the sequence. Then write 2 in the “terms” column and ask the students which number you should write in the “numbers” column. Ask them why you should put 5 in this column. Ask students what term should come next. Ask students which number is the third number in the sequence. Ask students where you should write this number.
- Ask students if they notice anything about the numbers Explain that the same thing is happening to each term to make the number. Tell students that they can think of the table as a machine. The term goes in the “terms” column. Something happens in the middle. Draw a squiggly line in between the words “terms” and “numbers” to represent the machine. Then explain that the number is what comes out after the term has been through the machine. Explain that the same thing is happening to each term in the middle.
- Ask students to look at the terms and look at the numbers and see if they can figure out what is happening in the “machine”. Most likely students will point out that the number 3 is being added to each term to make the number. Explain to

students that this means that the rule of this pattern is terms + 3 = numbers. Model writing the formula on the overhead.

- Repeat this process using the number pattern 6, 7, 8, 9.
- Next give students the number sequence 5, 6, 7 and ask them to represent it pictorially.
- Create a function table using 5, 6, 7 and ask students to help you fill it in. Ask them what they notice about the numbers and ask them if they can figure out the rule. Model writing the rule on the overhead.
- At this time students will need to break into cooperative groups.

Student Application-

- Distribute three strips of paper to each student.
- Ask each student to create their own growing number pattern using 4 terms and have them write it on their strip.
- Next have students switch number patterns with another member of their group.
- The students will create a function table in order to figure out the rule that their partner used in their pattern. Have the students write out the function table and the rule on their partner's strip.
- Have the students repeat this process two more times.

Embedded Assessment-

- Distribute SR4. Ask each student to complete the function table and to explain how they know that the numbers that they filled in are correct. Answers may be found on TR3.

Reteaching-

- Display the number pattern 2, 3, 4, 5.
- Model placing these numbers in a function table. Ask students to help you figure out the rule.

Extension-

- Explain to students that more than one operation can be used and that more than one number can be used in the rule.
- Ask students what they notice about the numbers. When they point out that one is added to each number to make the next number explain that you can use this to help you figure out the rule. Explain that since one is added to each number we know that -1 will be part of the rule.
- If your students are having trouble identifying the rule ask: We know that one operation included in the rule is subtraction so what do you think the other operation might be? Then you can have your students use guess and check to figure out the rule.
- Motivate students for tomorrow by asking them if they remember that by the end of this unit we will be able to figure out how many bumps are on the pineapple, how many segments are on the pinecone, and how many seeds are in the sunflower without counting them one by one.

- Remind them that today we learned about Fibonacci and you stated that Fibonacci sequence is connected to how you will figure this out. Ask the students how they think they will be able to solve the problem based on what they have learned about patterns. Ask them to think about this because answering this question is part of their homework for tonight. Distribute SR5 to be completed for homework. Answers may be found on TR4.

Lesson 3

Preassessment –

- Review SR5 and encourage students to give you the answers. Discuss any problems that the students struggled with. Ask for volunteers to explain how they thought they would solve the pineapple, pinecone, sunflower problem.
- Display the number pattern 3, 4, 5, 6 using overhead pattern blocks.
- Ask students to write the pattern in number form. After observing students, write the number sequence 3, 4, 5, 6 on the overhead.
- Model putting the numbers in a function table. Ask students to help you figure out the rule. Ask them if only one operation and number can be used in a rule.

Launch-

- Announce that today is the day they are going to solve the problem of how many bumps are on the pineapple, how many segments are on the pinecone, and how many seeds are in the sunflower.
- Point out that things in nature have growing patterns just like there are growing number patterns in mathematics.

Teacher Facilitation-

- Write Fibonacci number sequence 1, 1, 2, 3, 5, 8, 13, 21 on the overhead and identify it as so.
- Explain that the bumps on the pineapple, the segments on the pinecone, and the seeds in a sunflower follow this number sequence.
- Ask students to copy this number pattern using their snap cubes.
- Ask them to continue the pattern by adding the next term using their snap cubes.
- Ask the students what they notice about the pattern. Ask them if each number in the sequence increases or decreases by the same amount. Ask the students if they can identify the rule the number sequence follows.
- If students know the rule, emphasize the rule by writing the sum of the first two numbers above them and continue down the sequence.
  2, 3, 5, 8, 13, 21, 34
- Ex. 1, 1, 2, 3, 5, 8, 13, 21

- If students do not know the rule point out that the first two numbers added together equal the third number or that the two previous terms added together equal the next term.
- Show students your numbered pine cone. Refer to TR3 to help you number your pinecone correctly.

Student Application-

- Give each student a pinecone.
- Ask students to predict the number of segments they think are on the pinecone
- Record student predictions.
- Ask them to write the number 1 on the first segment at the top of their pine cone using a white out pen.
- Ask them to continue numbering the segments of the pinecone descending from the top moving clockwise until they have numbered each segment on the pinecone.
- Remind students that Fibonacci sequence can be found in nature.
- Ask students if anyone can point out how the segments on the pinecone follow the Fibonacci sequence.
- To elicit responses ask the students to look at the number of segments in each row.
- Ask for a volunteer to remind the class of the rule of Fibonacci sequence.
- Ask the students to look at the number of segments in the first row and the second row. Then ask them to add these two numbers. Next ask them how many segments are in the third row.
- The students should point out that there is one segment in the first row and two segments in the second row and three segments in the third row so the rule is adding the number of bumps in the two previous rows to get the number of bumps in the third row just like the rule of Fibonacci sequence is to add the two previous terms to get the next term.
- Ask students if they enjoyed writing all of those numbers on the pinecone or if they would like to think of a quicker way to figure out how many bumps are on the pineapple and how many seeds are in the sunflower.
- Ask students how they think we can do this. Most likely they will suggest that we can use Fibonacci sequence to help us.
- Ask the students to put the terms and numbers in Fibonacci sequence in a function table.
- See if students notice that the differences between the numbers recreate the sequence.
- Ask students if we can use the rule that the two previous terms added together equal the next term to determine what the 54^{th} term is.
- Ask students what the 100^{th} term is.
- Most likely they will point out that we can not figure out the 54^{th} term or the 100^{th} term because we do not know the 52^{nd} and 53^{rd} terms or the 98^{th} and 99^{th} terms.

- Hold up the pineapple and ask students how we can figure out how many bumps are on the pineapple. Refer to TR9 for help with determining the rows on the pineapple.
- Ask students to describe the pineapple again. Remind students that the segments on the pinecone were in rows and most likely they will point out that the bumps on the pineapple are in rows.
- Explain how the rows are vertical and diagonal on the pineapple.
- Ask students what they notice about the number of bumps in the vertical rows and the number of bumps in the horizontal rows.
- Students should notice that the bumps in all of the diagonal rows equal 13 and the number of bumps in all of the vertical rows equal 8.
- Ask the students how we could figure out how many bumps are on the pineapple without counting all of the bumps.
- Most likely they will see that they could either count all of the vertical rows and multiply that number by 8 or count all of the diagonal rows and multiply that number by 13.
- If you have enough pineapples, each group can determine the number of bumps on their pineapple. If not, the class as a whole can determine how many bumps are on the pineapple. All pineapples follow this pattern regardless of their size so it would be interesting for your students to experiment with pineapples of different sizes.
- After the students have determined the number of bumps on the pineapple hold up the sunflower and ask students to figure out how many seeds are in the sunflower.
- Use TR10 to help you see the spirals of seeds in the sunflower.
- The sunflower seeds are arranged in 34 spirals opening clockwise. The first spiral contains 3 seeds, the second spiral contains 5 seeds, the third spiral contains 8 seeds and so on.
- Ask the students if there are rows or spirals in the sunflower if no one points this out on their own.
- Point out the spirals to students and ask them how many seeds are in the first spiral, the third spiral, and so on.
- Ask students how they can figure out how many seeds are in the sunflower without counting all of them.
- Students should realize that they need to determine the number of seeds in the 32^{nd} and 33^{rd} spiral and add them together to determine the number of seeds in the entire sunflower.
- Again if you have enough sunflowers, each group can determine the number of seeds in their sunflower. If not, the class as a whole can determine how many seeds are in the sunflower.

Embedded Assessment-

- Use observation and questioning to evaluate whether or not students were able to identify the rule of Fibonacci sequence. You may ask questions such as: What number would come after 21 in the sequence?, Why are there two 1's in the

beginning of the sequence?, and What would you need to know to tell me the 15^{th} term in the sequence?

Reteaching-

- Show students the pattern 4, 5, 6, 7 using overhead pattern blocks.
- Ask the students to copy this pattern using their snap cubes.
- Have the students continue the pattern by adding the next number of snap cubes that would appear in the sequence.
- Ask the students to figure out the rule using a function table.
- Students should determine that the rule is term + 7 – 4 = number.

Extension-

- Ask students if they can think of anything else in nature that may follow Fibonacci sequence.
- If time allows have students study their numbered pinecones to find other patterns that the rows create. For example, the numbers on the segments increase by 8 going down the pinecone..

Summative Assessment:

Evaluate the Brief Constructed Response (SR6) to determine how well each student understood the unit. Use the MSA Rubric to grade student responses. Answers may be found on TR5.

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Predicting Patterns

Pattern Maker: _____

Pattern Solver: _____

Write the pattern in number sequence:

_____

_____

_____

Describe your partner's pattern:

_____

_____

_____

_____

_____

Predict and fill in the next number in the series:

_____

_____

_____

Brief Constructed Response-

Name: _____

Predict the next numbers in the pattern below:

5, 15, 25, 35, 45,____, _____, ____, _____

How were you able to determine the next numbers in the pattern? <u>Explain your answer.</u>

_____

_____

_____

_____

_____

Teacher Answer Sheet

Brief Constructed Response-

Name: _____

Predict the next number in the pattern below:

5, 15, 25, 35, 45, 55, 65, 75, 85

How were you able to determine the next numbers in the pattern? Explain your answer.

*** See MSA Rubric**

Homework – Lesson 1

Name: _____ Date: _____

Look at the pattern of numbers below:

7, 9, 11, 13

What is the next number in the pattern? Circle the correct answer.

a. 14
b. 15
c. 16
d. 17

Teacher Answer Sheet

Homework – Lesson 1

Name: _____ Date: _____

Look at the pattern of numbers below:

7, 9, 11, 13

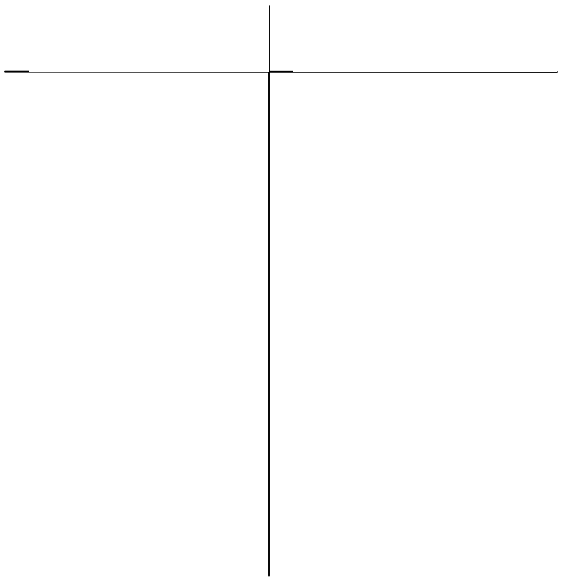
What is the next number in the pattern? Circle the correct answer.

e. 14
f. 15* (correct answer)**
g. 16
h. 17

Brief Constructed Response

Name: _____ Date: _____

Look at the function table below and fill in the missing numbers.



How did you determine the missing numbers? Explain your answer.

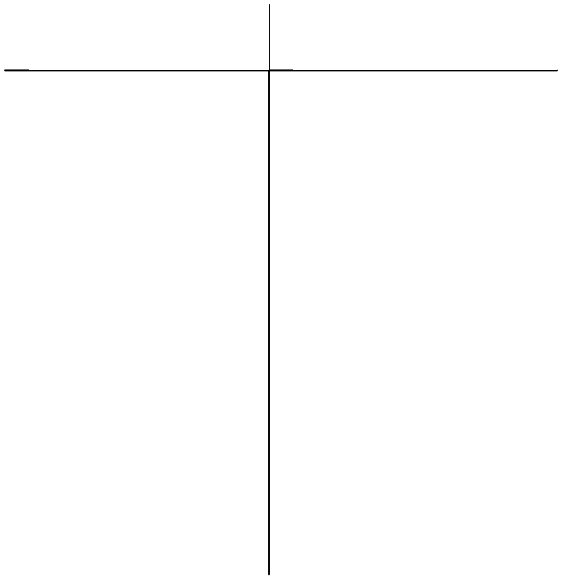
_____
_____
_____
_____
_____

Teacher Answer Sheet

Brief Constructed Response

Name: _____ Date: _____

Look at the function table below and fill in the missing numbers.

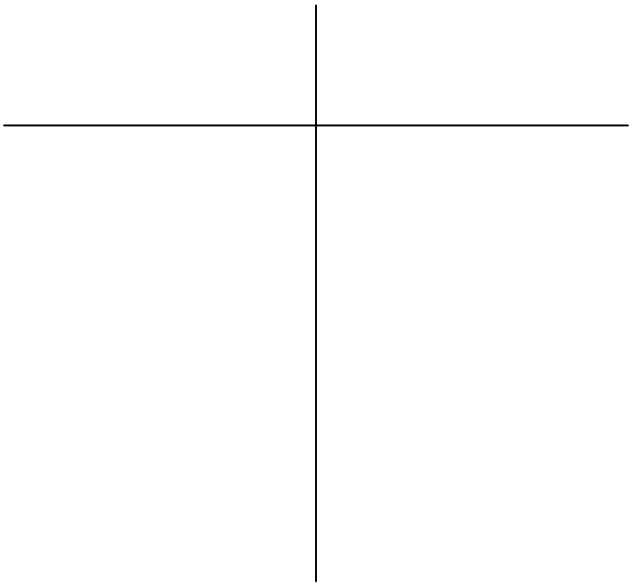


What method did you use to determine the missing numbers? Explain your answer.

*** See MSA Rubric**

Homework – Lesson 2

1. Look at the function table below.



2. What is the rule being applied in the function table? Circle the correct answer.

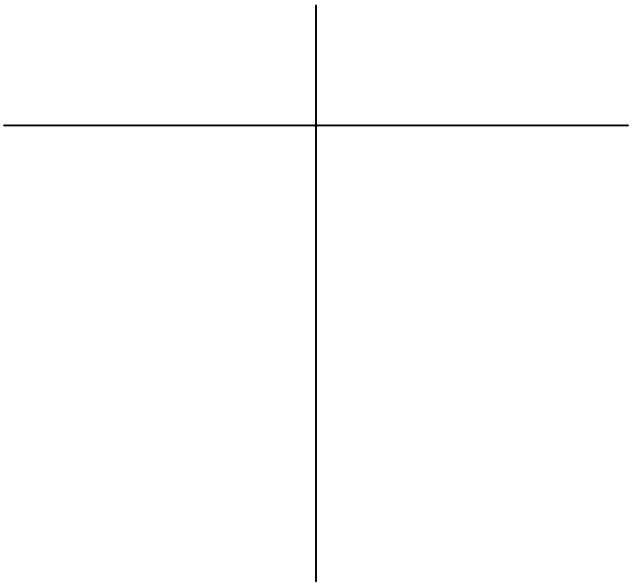
a. n + 16n
b. n + 5
c. 5n

3. What would be the output number for term 10?

a) 20
b) 16
c) 15
d) 11

Homework Answer Sheet
Homework – Lesson 2

1. Look at the function table below.



2. What is the rule being applied in the function table? Circle the correct answer.

d. n + 1 * (correct answer)**
e. 6n
f. n + 5
g. 5n

3. What would be the output number for term 10?

e) 20
f) 16
g) 15 * (correct answer)**
h) 11

Brief Constructed Response

Name_____ Date_____

Look at the number sequence below.

1 1 2 3 5 8 13 21

Part A

Write the next three numbers that would appear in this number sequence.

_____ _____ _____

Part B

Use what you know about growing patterns and Fibonacci sequence to explain why your answer is correct. Use vocabulary words in your explanation.

_____

_____

_____

_____

_____

Part A

Bridgette and Cedrick were each given a number sequence that they needed to find the rule for. They each decided to complete a function table to in order to find the rule. What rule did they use to complete their tables?

Bridgette's Table

Terms	Numbers
1	4
2	8
3	12
4	16

Cedrick's Table

Terms	Numbers
6	24
7	28
8	32
9	36

Rule:

Part B

Use what you know about function tables to explain why your answer is correct. Use words and/or numbers in your explanation.

_____

_____

_____

_____

_____

Brief Constructed Response

Name_____ Date_____

Look at the number sequence below.

1 1 2 3 5 8 13 21

Part A

Write the next three numbers that would appear in this number sequence.

34 55 89

Part B

Use what you know about growing patterns and Fibonacci sequence to explain why your answer is correct. Use vocabulary words in your explanation.

*Refer to the MSA Rubric (TR10)

Part A

Bridgette and Cedrick were each given a number sequence that they needed to find the rule for. They each decided to complete a function table to in order to find the rule. What rule did they use to complete their tables?

Bridgette's Table

Terms	Numbers
1	4
2	8
3	12
4	16

Cedrick's Table

Terms	Numbers
6	24
7	28
8	32
9	36

Rule: terms X 4 = numbers

Part B

Use what you know about function tables to explain why your answer is correct. Use words and/or numbers in your explanation.

* Refer to the MSA Rubric (TR10)

Building Patterns

Pattern that grows by 3:

Pattern that doubles the previous term:

Pattern that increases by 4:

Fibonacci, Leonardo Pisano

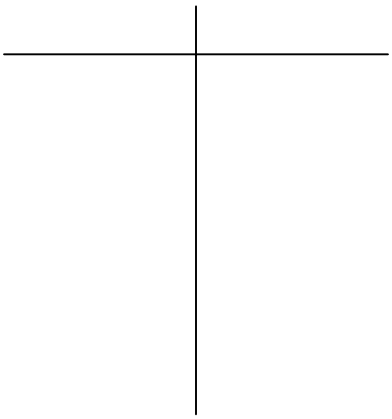
Born: 1170 probably in Pisa

Died: 1250

- **Leonardo Pisano, nicknamed Fibonacci, also used the name Bigollo, which means good-for-nothing.**
- **Fibonacci was born in Italy, but was educated in North Africa. He traveled widely with his father and received instruction in the school of accounting.**
- **He returned to Italy in 1200 A.D.**
- **He lived in the days before printing, so his books were hand written and the only way to have a copy of one was to have another hand written copy made.**
- **He was a sophisticated mathematician and his achievements were clearly recognized but it was his practical application, like "growing patterns and prime numbers," which made him famous.**
- **His most famous writing was a book called Liber abbaci, published in 1202. This book was based on arithmetic and algebra.**

- **The second section of Liber abbaci contained problems like**: *A certain man put a pair of rabbits in a place surrounded on all sides by a wall. How many pairs of rabbits can be produced from that pair in a year if it is supposed that every month each pair begets a new pair which from the second month on becomes productive?*

Example

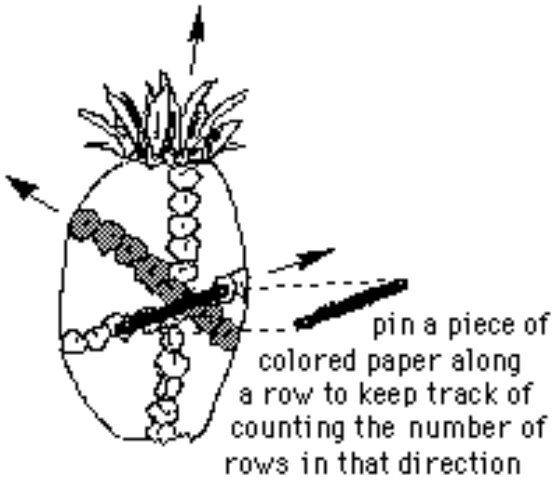


- **The third section in Liber abbaci led to the introduction of the Fibonacci numbers and the Fibonacci sequence for which he is best remembered.**

http://www.history.mcs.st- andrew.ac.uk/history/mathmeticians/Fibonacci/html

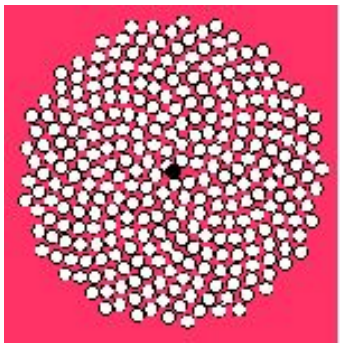
Fibonacci Numbers:
1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144

Pineapple Rows



Found @ http://www.shout.net/~mathman/html/prob7.html

Sunflower Spirals



Found @ http://www.pims.math.ca/education/2000/bus00/sunflower/

MSA Mathematics BCR Rubric
Grades 3 through 8

2 The response demonstrates a complete understanding and analysis of a problem.

- Application of a reasonable strategy in the context of the problem is indicated.
- Explanation[1] of and/or justification[2] for the mathematical process(es) used to solve a problem is clear, developed, and logical.
- Connections and/or extensions made within mathematics or outside of mathematics are clear.
- Supportive information and/or numbers are provided as appropriate. [3]

1 The response demonstrates a minimal understanding and analysis of a problem.

- Partial application of a strategy in the context of the problem is indicated.
- Explanation[1] of and/or justification[2] for the mathematical process(es) used to solve a problem is partially developed, logically flawed, or missing.
- Connections and/or extensions made within mathematics or outside of mathematics are partial or overly general, or flawed.
- Supportive information and/or numbers may or may not be provided as appropriate. [3]

0 The response is completely incorrect, irrelevant to the problem, or missing. [4]

Notes:

[1] **Explanation** refers to students' ability to communicate **how** they arrived at the solution for an item using the language of mathematics.

[2] **Justification** refers to students' ability to support the reasoning used to solve a problem, or to demonstrate **why** the solution is correct using mathematical concepts and principles.

[3] Students need to complete rubric criteria for ***explanation, justification, connections*** and/or ***explanation*** as cued for in a given problem.

[4] An exact copy or paraphrase of the problem that provides no new relevant information will receive a score of "0".